DISPERSION OF A PLASMA CLOUD IN A UNIFORM MAGNETIC FIELD

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The numerical solution of the two-dimensional gasdynamical problem of the dispersion of a plasma cloud in a magnetic field which is uniform to infinity is described. The disturbance of the field and the deformation of the cloud are taken into account self-consistently.

The conditions of stability of such a flow are examined.

1. In experiments of the Argus type [1] (i.e., during nuclear explosions of low power at altitudes of 500 km) a plasma cloud is formed which disperses with a mean velocity of 100 km/sec. The question of retardation and energy conversions during the expansion of such a cloud into empty space in which there is a uniform magnetic field has been examined in [2-4]. The two-dimensional nature of this problem was taken into account approximately in these works.

The results of a numerical solution of the two-dimensional, axially symmetrical problem of the dispersion of a plasma cloud in a uniform magnetic field are presented below. The motion of the cloud is described by equations of gasdynamics; the magnetic field pressure $P = H^2/8\pi$ is given as the boundary condition. The variation in the magnetic field outside the cloud is described in a qausistationary approximation [5]. In accordance with this approximation the Laplace equation was solved for the magnetic field potential φ($H = -\text{grad}\ \varphi$) with a "superconducting" boundary condition at the boundary of the cloud

$$\begin{gathered}\Delta\varphi = 0 \\ (\mathbf{n}\nabla\varphi)_{S(t)} = 0, \qquad \varphi|_{r\to\infty} = 0\end{gathered} \tag{1.1}$$

where S(t) is the surface which determines the boundary of the plasma cloud at the time t; **n** is the normal to the surface S(t).

The self-similar solution of the problem of the dispersion of a gas sphere into a vacuum [6] was used as the initial gasdynamical values (1.2)

$$\begin{gathered}\rho(r, 0) = \rho_0 \\ p(r, 0) = p_0(1 - r^2/R_0^2), \qquad 0 \leqslant r \leqslant R_0 \\ v_r(r, 0) = v_0(r/R_0)\end{gathered} \tag{1.2}$$

The initial distribution of φ outside the sphere was taken from the solution of the problem of the disturbance of a magnetic field outside a superconducting sphere placed in the uniform magnetic field [7]

$$\varphi = (\mathbf{H}_0\mathbf{r})\,(1 + {}^1/_2\, R_0^3/r^3) \tag{1.3}$$

2. The system of equations of gasdynamics with the initial condition (1.2) and the boundary conditions $P|_{S(t)} = H^2/8\pi$ was solved by the method of [8]. The Laplace equation (1.1) was solved at each time step by the gasdynamical method of determination of [9]. The solution of the nonstationary problem

$$\frac{1}{\varkappa}\frac{\partial\varphi}{\partial\tau} = \Delta\varphi, \quad (\mathbf{n}\varphi)|_{S(t)} = 0, \quad \varphi|_{r\to\infty} = 0 \tag{2.1}$$

with large values of the thermal conductivity coefficient $\varkappa$ = const converges to the solution of the Laplace equation (1.1).

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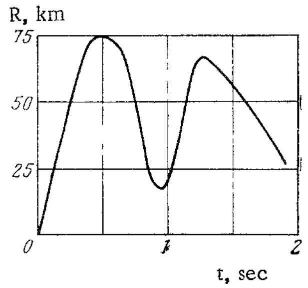

Fig. 1

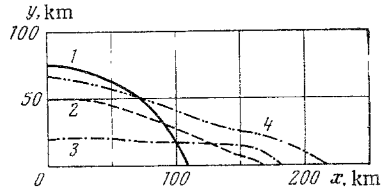

Fig. 2

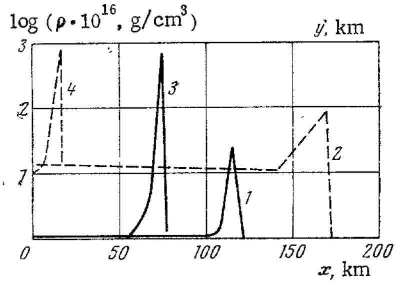

Fig. 3

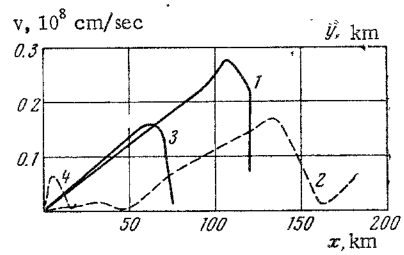

Fig. 4

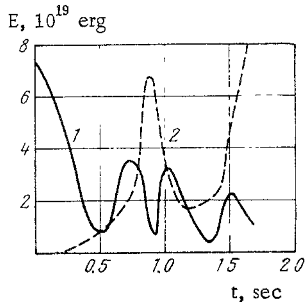

Fig. 5

The initial parameters of the plasma cloud taken in the calculation were the density $\rho_0 = 0.24 \cdot 10^{-9}$ g/cm^3, initial cloud radius $R_0 = 10^5$ cm, and velocity of the boundary $v_0 = 0.32 \cdot 10^8$ cm/sec. The plasma cloud was retarded by a magnetic field with intensity $H_0 = 0.5$ Oe.

The calculations conducted showed that the allowance for the two-dimensional nature of the flow leads to motion of the plasma cloud boundary which was not considered in [2, 3]. This motion is characterized by the fact that compression of the plasma cloud develops after the retardation of the cloud in the direction perpendicular to the undisturbed magnetic field. The spreading starts again from the time $t \approx 1$ sec. The subsequent variation in the size of the cloud in the direction perpendicular to H_0 has an oscillatory nature with a decreasing amplitude (see Fig. 1). The shape of the plasma cloud boundary at different times is present in Fig. 2 (curve 1: t = 0.35, 2: 0.69, 3: 0.9, 4: 1.31 sec). The x axis coincides with the direction of the undisturbed field.

Profiles of the microscopic characteristics of the plasma at different times are presented in Figs. 3-4. Curves 1 and 2 are profiles of the values along the x axis at the times t = 0.4 sec (curve 1) and t = 0.9 sec (curve 2). Curves 3 and 4 are profiles of the values along the y axis at the times t = 0.4 sec (curve 3) and t = 0.9 sec (curve 4). The radial density distribution proves to be quite nonuniform. In 0.1 sec a "crust" forms near the cloud boundary which has a thickness of 1 km and a mean particle concentration of 10^8-10^9 cm^{-3}.

The integral energy characteristics of the dispersing cloud at different times are presented in Fig. 5 (curve 1: kinetic energy of cloud, curve 2: internal energy of cloud). For a complete analysis of the energy balance it is necessary to make allowance in the problem for electromagnetic radiation in the wave zone. Such a problem is rather complicated, however, because of the presence of two considerably different characteristic velocities.

3. For an estimate of the limits of stability of such flow we will use the following model. A homogeneous plasma occupies a half-space and moves with constant acceleration **a** perpendicular to a uniform magnetic field H_0 which is displaced by the moving plasma. This model is applicable to the case under study if one considers disturbances of the boundary with characteristic wavelengths $\lambda \sim 2\pi/k \ll L$ (where L is the crust thickness) and the acceleration is $\sim \Delta v/\Delta t \sim 10^8$ cm/sec^2 (see Fig. 1). It is obvious that the pattern of development of an instability in this model is analogous to the appearance of a gutter instability

when a plasma is confined in a uniform gravitational field [10]. A gutter instability develops with an increment which is determined from the solution of the following dispersion equation:

$$\omega^2 = -|k|a + (\mathbf{k}\mathbf{v}_A)^2, \quad v_A = H/\sqrt{4\pi\rho} \tag{3.1}$$

where ρ is the density of the moving plasma.

It follows from (3.1) that the plasma motion under consideration is most unstable relative to the formation of oscillations with $\mathbf{k} \perp \mathbf{H}_0$. The rise time of oscillations with $\lambda \sim L = 10^5$ cm is $\sim 10^{-1}$ sec. This estimate shows the need for a more detailed analysis of the problem of the development of a gutter instability in the problem under consideration.

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